# Half life and radioactive equilibrium

## Curricular Relevance

<table>
<thead>
<tr>
<th>Area of Expertise:</th>
<th>Physics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education Level:</td>
<td>University</td>
</tr>
<tr>
<td>Topic:</td>
<td>Modern Physics</td>
</tr>
<tr>
<td>Subtopic:</td>
<td>Nuclear Physics - Radioactivity</td>
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<tr>
<td>Experiment:</td>
<td>Half life and radioactive equilibrium</td>
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</tbody>
</table>

### Difficulty

- " difficile

### Preparation Time

- 1 Hour

### Execution Time

- 2 Hours

### Recommended Group Size

- 2 Students

### Additional Requirements:

### Experiment Variations:

### Keywords:

- parent substance, daughter substance, rate of decay, disintegration or decay constant, counting rate, half life, disintegration product

## Overview

### Short description

### Principle

The half-life of a $^{137m}$Ba daughter substance eluted (washed) out of a $^{137}$Cs isotope generator is measured directly and is also determined from the increase in activity after elution.

![Experimental set-up](image-url)
### Equipment

<table>
<thead>
<tr>
<th>Position No.</th>
<th>Material</th>
<th>Order No.</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Isotope generator Cs-137, 370 kBq</td>
<td>09047-60</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Geiger-Mueller counter</td>
<td>13606-99</td>
<td>1</td>
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<td>3</td>
<td>Geiger-Mueller counter tube, 15 mm (type B)</td>
<td>09005-00</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Glass beaker DURAN®, short, 250 ml</td>
<td>36013-00</td>
<td>2</td>
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<tr>
<td>5</td>
<td>Test tubes 100x12 mm, FIOLOX, 100pc</td>
<td>36307-10</td>
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<td>6</td>
<td>Rubber stopper, d=14.5/10.5 mm, w/o</td>
<td>39253-00</td>
<td>1</td>
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<tr>
<td>7</td>
<td>Base plate for radioactivity</td>
<td>09200-00</td>
<td>1</td>
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<tr>
<td>8</td>
<td>Counter tube holder on fixating magnet</td>
<td>09201-00</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>Plate holder on fixating magnet</td>
<td>09203-00</td>
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<tr>
<td>10</td>
<td>Source holder on fixating magnet</td>
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</table>

### Tasks

1. Measure the activity of the isotope generator as a function of time immediately after elution.
2. Measure the activity of a freshly eluted solution of $^{137m}\text{Ba}$ as a function of time.

### Set-up and procedure

Set up the experiment as shown in Fig. 1.

**Procedure**

In accordance with the instructions the isotope generator is eluted into a glass beaker which should then be placed as far away from the counter tube as possible. Make a U-shaped cap from a strip of aluminium sheet and put it over the tube: it will absorb the electrons in the betadecay phase which would otherwise interfere with the experiment. To measure the increase in activity it is advisable to read off the impulse rate every 30 seconds after elution (counting ratemeter time constant = 10 seconds). Activity and impulse count are sufficiently proportional at low impulse rates.

To measure the half-life of the $^{137m}\text{Ba}$ isotope, first elute the isotope generator in a test tube, then place it as far away from the rest of the equipment as possible. The counter tube (without the aluminium cap) can now be set up immediately in front of the bottom end of the test tube.

### Results

The isotope generator contains 400 kBq of $^{137}\text{Cs}$, which serves as the parent substance; its half-life is 30.25 years. $^{137}\text{Cs}$ decays into the barium isotope $^{137}\text{Ba}$ with the emission of $\beta$-radiation. This transi-tion occurs, in part, directly (approximately 5%) to the stable ground state of $^{137}\text{Ba}$ and, in part (approximately 95%) via the meta stable state of $^{137m}\text{Ba}$. $^{137m}\text{Ba}$ decays with a half-life of only 2.6 min under emission of $\gamma$-radiation ($E_\gamma = 662 \text{keV}$) in the stable ground state of $^{137}\text{Ba}$. The decay chain is thus

$$ N_1 \rightarrow N_2 \rightarrow N_3 $$

where

$N_1 = \text{number of } ^{137}\text{Cs atoms}$

$N_2 = \text{number of } ^{137m}\text{Ba atoms}$

$N_3 = \text{number of } ^{137}\text{Ba atoms}$

$\lambda_1 = \text{disintegration constant of } ^{137}\text{Cs}$

$\lambda_2 = \text{disintegration constant of } ^{137m}\text{Ba}$

The rates of disintegration are:

$$ \frac{dN_1}{dt} = -\lambda_1 N_1 $$

$$ \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 $$

Solving the system of two differential equations gives for $N_2(t)$, with the initial condition that $N_2(t = 0) = 0$:
The activity of the daughter substance is thus

\[ A_2(t) = \lambda_2 N_2 = A_1(0) \frac{\lambda_2}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \]

As the half-life of the parent substance \( T_{1/2}(1) \) is much greater than that of the daughter substance \( T_{1/2}(2) \) in this case, then \( \lambda_1 \ll \lambda_2 \). If we now neglect \( \lambda_1 \) in relation to \( \lambda_2 \), we obtain

\[ A_2(t) = A_1 (1 - e^{-\lambda_2 t}) \]

with \( A_1 \) constant.

When \( t \) is very long,

\[ A_2(t) - A_1 (t) \]

and

\[ \lambda_2 N_2 = \lambda_1 N_1 \]

i.e. the substances are in equilibrium (dynamic equilibrium, steady state).

If this equilibrium is disturbed by the removal of the daughter substance, the system will try to restore the equilibrium by forming more of it. For the increase in the daughter substance, we have:

\[ A_2(t) = A_1 (1 - e^{-\lambda_2 t}) \]

with \( A_1 \) constant.

If the equilibrium activity

\[ A_2(t = \infty) = A_1 \]

Is measured prior to elution, then \( \lambda_2 \) can be calculated from the measured values \( A - A(t) \) because

\[ A_1 - A_2(t) = A_1 (e^{-\lambda_2 t}) \]

In the elution process, the \(^{137}\text{m} \text{Ba}\) is washed out of the isotope generator. Figure 2 shows the counting rate as a function of time. A counting rate that decreases exponentially over time results. The logarithmic counting rate is plotted as a function of time, the straight line given in Fig. 3 is obtained.
In this case, the counting rate decreases to half of its original value after a time period $\tau = 153.69\ s$. Thus, it follows that:

$$\dot{N} = \dot{N}_0 e^{-\lambda t}$$

and

$$\frac{1}{2} \dot{N} = \dot{N}_0 e^{-\lambda \tau}$$

where

$\dot{N}$ Actual rate of decay
$\dot{N}_0$ Counting rate at time
$\lambda$ Decay constant
$t$ Time
$\tau$ Half-life

Thus,

$$\lambda = \frac{\ln 2}{\tau}.$$ 

By taking the logarithm of

$$\dot{N} = \dot{N}_0 e^{-\lambda t}$$

the following is obtained

$$\ln \dot{N} = -\lambda \ln \dot{N}_0$$

Comparing the coefficients with the regression line

$y = mtb$

shows that the slope of the regression line $m$ corresponds to the negative decay constant

$m = -\lambda$

The decay constant $\lambda$ is characteristic for the gradual decline in radioactivity: It is thus linked to the specific time period in which the substance quantity decays to half of its initial radioactivity (half-life $\tau$).

**Formation of daughter substance subsequent to disturbance of the equilibrium by elution**

Calculation of the half-life. At time $t$ an activity is measured that can be calculated according to the following formula:

$$\dot{N}(t) = \dot{N} + \dot{N}_0 (1 - e^{-\lambda t})$$

where
\( \tilde{N}(t) \) Actual counting rate

\( \tilde{N} \) Activity of the \( \beta \)-radiating \( ^{137} \text{Cs} \) (through the opining in the source) and the residual activity of the \( ^{137m} \text{Ba} \), which results from elution.

\( N_0 \) Equilibrium activity of the daughter substance, \( ^{137m} \text{Ba} \).

By subtracting the equilibrium activity after approximately 650 s (which is comprised of \( \tilde{N} \) and \( N_0 \), from the respective actual counting rate), in this case 504.28 (cf. Fig. 7) and calculating the natural logarithm of this variable’s value, the diagram in Fig. 8 is obtained. The half-life for this measurement, \( \tau = 142.33 \text{ s} \), follows from the slope of the drawn regression line

\[
\tau = \frac{\ln 2}{m} = \frac{\ln 2}{-0.00457} = 142.33 \text{ s}
\]

**Remarks**

The calculation shown above can be performed using the \texttt{ln(abs(x-504.28))}. When using radioactive substances, conform absolutely to the stipulations of the respective applicable radiation protection regulations. Radioactive substances can be hazardous to your health! Always reduce the time spent handling radioactive substances to a minimum. Do not eat or drink anything in the presence of radioactive substances and always wash your hands after contact with them!